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The moduli dependency of refined topological amplitudes is being addressed.

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Abstract

With the aim of providing a worldsheet description of the refined topological string, we continue the study of a particular class of higher derivative couplings $F_{g,n}$ in the type II string effective action compactified on a Calabi–Yau threefold. We analyse first order differential equations in the anti-holomorphic moduli of the theory, which relate the $F_{g,n}$to other component couplings. From the point of view of the topological theory, these equations describe the contribution of non-physical states to twisted correlation functions and encode an obstruction for interpreting the $F_{g,n}$ as the free energy of the refined topological string theory. We investigate possibilities of lifting this obstruction by formulating conditions on the moduli dependence under which the differential equations simplify and take the form of generalised holomorphic anomaly equations. We further test this approach against explicit calculations in the dual heterotic theory.

Introduction

Since the first construction of topological string theory [1], its connection to higher derivative couplings in the string effective action has been a very active and fruitful field of study. Indeed, in [2], a series of higher loop scattering amplitudes Fg, in type II string theory compactified on a Calabi–Yau threefold, was computed and shown to capture the genus g free energy of the topological string. These couplings are BPS protected and involve 2g chiral supergravitymultiplets. The result of [2] is interesting from a number of different perspectives. On the one hand, the Fg encode very important target space physics, for example in computing macroscopic corrections to the entropy of supersymmetric black holes (see for example [3]). On the other hand, they provide a concrete worldsheet description of the topological string which is very powerful in studying its properties [4].

During the last two decades, the work of [2] has been extended and many new relations between topological correlation functions and higher derivative effective couplings in string theory have been found [5–13]. Along these lines, it was suggested in [14] that a suitable generalisation Fg,n of the Fg could provide a worldsheet description of the refined topological string. The refinement of the topological string consists of a one-parameter deformation of topological string theory, inspired by recent progress in the study of supersymmetric gauge theories [15–17], so that its point-particle limit reproduces the partition function of supersymmetric gauge theories in the full -background. In this correspondence, the topological string

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coupling gs is identified with one of the geometric deformation parameters −, while the refinement is an extension associated to the second parameter +. The first proposal successfully satisfying this requirement was presented in [18], through explicit computations to all orders in α in heterotic string theory.
From the target space point of view, numerous different descriptions of the refinement exist, such as the counting of particular BPS-states in M-theory [19], the refined topological vertex [20], matrix models using refined ensembles [21] or through a construction of the -background using the so-called flux-trap background [22]. In a recent work [18], we proposed a worldsheet description of the refined topologic string using a generalisation of the couplings Fg involving two Riemann tensors and 2g − 2 insertions of graviphoton field strengths, by additional insertions of chiral projections of specific vector multiplets. These couplings are of the general form discussed in [23,14] (see also [2,8,11–13]). The precise nature of the additional insertions is crucial in exactly reproducing the Nekrasov partition function both perturbatively [18] and nonperturbatively [24]. Specifically, working in heterotic string theory compactified on K3 × T 2, we computed in [18] a series of refined couplings FT¯ g,n which include additional 2n insertions of the field strength tensor of the vector superpartner of the Kähler modulus of T 2 (T¯-vector). These amplitudes are exact to all orders in α and start receiving corrections at the one-loop level in gs. At a particular point of enhanced gauge symmetry in the heterotic moduli space, they reproduce exactly the perturbative part of the Nekrasov partition function in the point particle limit for arbitrary values of the deformation parameters.1 A very strong check of our proposal was performed in [24] (see [26,27] for reviews and [28,29] for earlier partial results) by computing gauge theory instanton corrections to Fg,n, which precisely reproduce also the non-perturbative part of the gauge theory partition function.
The connection between the couplings studied in [18] and the full Nekrasov partition function is a very strong hint that our proposal for the FT¯ g,n can indeed furnish a worldsheet descriptionof the refined topological string. In this context, non-physical states of the topological theory are required to decouple from Fg,n. In the unrefined case (i.e. for n = 0), this requirement has first been studied in [4]: in the twisted theory, the BRST operator is identified with one of the supercharges of the original N = 2 worldsheetsuperconformal theory. Thus, some of the moduli of the untwisted theory are not part of the topological BRST cohomology and are 'unphysical' from the latter point of view. This implies that Fg should possess holomorphy properties. In the supergravity formulation, this agrees with the fact that the Fg only depend on the chiral vector multiplet moduli and can be written in the form of BPS-saturated F-terms in R4|8 superspace. However, as pointed out in [4], in string theory, there is a residual dependence on the anti-holomorphic moduli due to boundary effects in the moduli space of the higher genus worldsheet. This gives rise to a recursive differential equation known as the holomorphic anomaly equation, which relates the anti-holomorphic moduli derivative of Fg to combinations of (holomorphic derivatives of) Fg with g < g.

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In this paper we study the question of the decoupling of anti-holomorphic moduli in the case of the Fg,n studied in [14,18] by deriving differential equations for the corresponding effective couplings. For n > 0, the Fg,n are no longer F-terms, but also contain chiral projections of superfields. Therefore, a priori, there are no constraints on their dependence on anti-holomorphic moduli, even at the level of supergravity. However, by analysing the structure of the couplings in superspace, we obtain differential equations which relate anti-holomorphic derivatives of Fg,n to new component couplings, and the latter can be realised as scattering amplitudes in string theory.

By studying these relations in detail in supergravity, we can reformulate the vanishing of the anti-holomorphic vector multiplet dependence in Fg,n as well-defined conditions on the moduli dependence of particular coupling functions in the effective action. The latter conditions go beyond the constraints of N = 2 supersymmetry and might be interpreted as a consequence of a U(1) isometry present in a special region in the string moduli space, as required from the point of view of gauge theory in order to formulate a supersymmetric -background [15–17]. In this case, since such isometries are generically not present in compact Calabi–Yauthreefolds, the conditions for decoupling the anti-holomorphic vector multiplets might be regarded as Ward identities related to the appearance of U(1) isometries in suitable decompactification limits.

Extending the supergravity analysis, we derive explicit differential equations for the Fg,n in the framework of the fully-fledged type II string theory compactified on generic Calabi–Yauthreefolds. We relate all new component couplings involved in these relations in the form of higher genus scattering amplitudes and express them as twisted worldsheetcorrelators on a genus g Riemann surface with 2n punctures. The equations we obtain contain corrections induced by boundary effects in the moduli space of the higher genus worldsheet. From the string theory perspective, the decoupling of non-holomorphic moduli translates into well-defined conditions on the worldsheetcorrelators. The upshot of our approach is that it provides a solid framework, based on physical string couplings, in which the above mentioned Ward identities may be analysed in the full worldsheet theory. In particular, we can formulate conditions under which the string-derived differential equations reduce to the recursive structure of a generalised holomorphic anomaly equation. Equations of this type were postulated in [30,31] as the definition of the refined topological string.

Finally, we also study the differential equations in the dual setup of heterotic string theory on K3 × T 2. On the heterotic side, the FT¯ g,n start receiving contributions at the one-loop level and therefore constitute the ideal testing-ground for the ideas developed in type II, particularly for certain decompactification limits. We find that in the large volume limit of T 2, they satisfy recursive differential equations which precisely match with the weak coupling version of our differential equations in type II, hence providing a non-trivial check of our approach. On the other hand, we use the heterotic setup to study boundary conditions to the differential equations developed in this work. Indeed, in [30,31], the field theory limit was used as a

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boundary condition to solve for the couplings Fg,n. In the present case, while the equations in type II are essentially covariant with respect to the choice of vector multiplet insertion in Fg,n, only the specific choice of the T¯-vector for FT¯ g,n was found in [18] to reproduce the gauge theory partition function. Here, we show that also other choices of vector multiplet insertions lead to the same boundary conditions when expanded around an appropriate point of enhanced gauge symmetry in the heterotic moduli space.

The paper is organised as follows. In Section 2, we prepare the ground by discussing the effective action couplings Fg,n and extract several relations among them implied by supersymmetry. In Section 3, we derive equations in type II string theory compactified on a Calabi–Yau threefold. We derive all necessary amplitudes at higher genus and identify string theoretic corrections to the supergravity equations as boundary terms of the worldsheet moduli space. In Section 4, we discuss simplifications of the differential equations which we propose to be the effect of U(1) isometries of the target space Calabi–Yau threefold. In particular, we point out that, under certain conditions, a recursive structure emerges in the equations, both at the supergravity and at the full string level in type II. In Section 5, we consider the dual heterotic theory on K3 × T 2. We first perform a check of the results obtained in type II from the heterotic dual computation and then provide boundary conditions to the differential equations by reproducing the Nekrasov partition function for different vector multiplet insertions in Fg,n. Finally, Section 6 contains a summary of our results and our conclusions. Several technical results are compiled in three appendices.

Interpretation and conclusions

In this paper, we have discussed the class of superspace couplings (2.7) in the N = 2 supergravity action. We have analysed consistency conditions between its various component terms that are imposed by supersymmetry. These do not simply constrain the moduli dependence of a single component coupling (e.g. holomorphicity as in the case of n = 0, see [2]), but rather relate different component terms with one another. These relations were formulated as first order differential equations, e.g. (2.16) and (2.17).

Based on the evidence in support of our proposal [18] for the Fg,n as candidates for the refinement of the topological string, following [14], we derived all couplings (2.13) as higher loop scattering amplitudes in the framework of type II string theory on a (compact) Calabi–Yau manifold. These string effective couplings were shown to satisfy (2.16) and (2.17) up to additional terms which arose as boundary contributions of the moduli space of the genus g worldsheet with n punctures. The latter play a similar role as the holomorphic anomaly found in [2] in the case of n = 0. The resulting equations (3.18) and (3.19) are solely a consequence of the N = (2, 2) worldsheetsupersymmetry and hold at a generic point in the string moduli space. Provided certain well-defined conditions are met, these equations reduce to a form involving only one type of component couplings and exhibit a recursive structure in both g and n. The resulting equation (4.8) is structurally similar to the generalised holomorphic anomaly equation proposed in [30,31] as a definition for the free energy of

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the refined topological string on local/non-compact Calabi–Yau manifolds.

These results support our proposal [18,24] for the couplings Fg,n as a worldsheet definition of the refined topological string. The present work further analyses the necessary conditions for the validity of our proposal. At a generic point in the moduli space of a (compact) Calabi–Yau manifold, the couplings Fg,n are not BPS-saturated and their (twisted) worldsheet representation (3.3) is not topological. This manifestsitself in the fact that the Fg,n are related to different classes of couplings. We expect that the U(1) isometry, recovered at certain regions in the boundary of moduli space, is responsible for a simplification of these equations (see e.g. (4.8)) that is appropriate for a topological object. We have provided the well-posed necessary and sufficient conditions (4.9) (formulated in terms of physical quantities only) for this modification to happen. Furthermore, by analysing the explicit form of the Fg,n in the dual heterotic theory on K3 × T 2, we obtained perfect agreement with the weak coupling limit of (4.8). An interesting open question concerns the study of explicit examples of Calabi–Yau geometries and the analysis of the geometric implications of the consistency conditions derived in this work.

As was also noted in [30,31], the differential equations are not sufficient to define the partition function of the free energy of the topological string since it must be supplemented by suitable boundary conditions. One such condition is the point particle limit in which the topological free energy, when expanded around a point of enhanced gauge symmetry, should reproduce the partition function for N = 2 supersymmetric gauge theories in a general -background. In the case of the string couplings Fg,n, this limit was analysed perturbatively and non-perturbatively in [18, 24] for A μ being identified with the vector superpartner of the heterotic T¯-modulus of T 2, and indeed the full gauge theory partition function was reproduced. In this work we have extended this analysis and found that all couplings Fg,n with $F_{g,n}$ with $\phi_* \in \frac{O(2,10)}{O(2)\times O(10)}$ reproduce perturbativelyNekrasov's partition function, when expanded around an appropriate point of enhanced gauge symmetry in the string moduli space.

In summary, the findings of this paper further corroborate our proposal that the string scattering amplitudes Fg,n can provide a worldsheet description of the refined topological string. Indeed, we have elucidated the conditions under which such an identification is possible. We have also shown that our proposal is compatible with other approaches towards the refined topological string. In particular, starting only from physical quantities (i.e. string scattering amplitudes), we have proposed a way of finding a generalised holomorphic anomaly equation, which e.g. in [30,31] was postulated as the definition of the refined topological string.

References

[1] E. Witten, Topological sigma models, Commun. Math. Phys. 118 (1988) 411; E. Witten, On the structure of the topological phase of two-dimensional gravity, Nucl. Phys. B 340 (1990) 281.

[2] I. Antoniadis, E. Gava, K.S. Narain, T.R. Taylor, Topological amplitudes in string theory, Nucl. Phys. B 413 (1994) 162, arXiv:hep-th/9307158.

[3] G. Lopes Cardoso, B. de Wit, T. Mohaupt, Corrections to macroscopic

Available online at www.jcsonline.in
Journal of Current Science & Humanities
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supersymmetric black-hole entropy, Phys. Lett. B 451 (1999) 309, arXiv:hep-th/9812082; G. Lopes Cardoso, B. de Wit, T. Mohaupt, Deviations from the area law for supersymmetric black holes, Fortschr. Phys. 48 (2000) 49, arXiv:hep-th/9904005; G. Lopes Cardoso, B. deWit, T. Mohaupt, Macroscopic entropy formulae and nonholomorphic corrections for supersymmetric black holes, Nucl. Phys. B 567 (2000) 87, arXiv:hep-th/9906094; G. Lopes Cardoso, B. de Wit, T. Mohaupt, Area law corrections from state counting and supergravity, Class. Quantum Gravity 17 (2000) 1007, arXiv:hep-th/9910179; T. Mohaupt, Black hole entropy, special geometry and strings, Fortschr. Phys. 49 (2001) 3, arXiv:hep-th/0007195.

[4] M. Bershadsky, S. Cecotti, H. Ooguri, C. Vafa, Kodaira–Spencer theory of gravity and exact results for quantum string amplitudes, Commun.Math. Phys. 165 (1994) 311, arXiv:hep-th/9309140.

[5] N. Berkovits, C. Vafa, N = 4 topological strings, Nucl. Phys. B 433 (1995) 123, arXiv:hep-th/9407190.

[6] N. Berkovits, C. Vafa, Type IIB R4 H∗(4g − 4) conjectures, Nucl. Phys. B 533 (1998) 181, arXiv:hep-th/9803145.

[7] H. Ooguri, C. Vafa, All loop N = 2 string amplitudes, Nucl. Phys. B 451 (1995) 121, arXiv:hep-th/9505183.

[8] I. Antoniadis, E. Gava, K.S. Narain, T.R. Taylor, Topological amplitudes in heterotic superstring theory, Nucl. Phys. B 476 (1996) 133, arXiv:hep-th/9604077.

[9] I. Antoniadis, K.S. Narain, T.R. Taylor, Open string topological amplitudes and gaugino masses, Nucl. Phys. B 729 (2005) 235, arXiv:hep-th/0507244.

[10] I. Antoniadis, S. Hohenegger, K.S. Narain, N = 4 topological amplitudes and string effective action, Nucl. Phys. B 771 (2007) 40, arXiv:hep-th/0610258.

[11] I. Antoniadis, S. Hohenegger, K.S. Narain, E. Sokatchev, Harmonicity in N = 4 supersymmetry and its quantum anomaly, Nucl. Phys. B 794 (2008) 348, arXiv:0708.0482 [hep-th].

[12] I. Antoniadis, S. Hohenegger, K.S. Narain, E. Sokatchev, A new class of N = 2 topological amplitudes, Nucl. Phys. B 823 (2009) 448, arXiv:0905.3629 [hep-th].

[13] I. Antoniadis, S. Hohenegger, K.S. Narain, E. Sokatchev, Generalized N = 2 topological amplitudes and holomorphic anomaly equation, Nucl. Phys. B 856 (2012) 360, arXiv:1107.0303 [hep-th].

[14] I. Antoniadis, S. Hohenegger, K.S. Narain, T.R. Taylor, Deformed topological partition function and Nekrasov backgrounds, Nucl. Phys. B 838 (2010) 253, arXiv:1003.2832 [hep-th].

[15] G.W. Moore, N. Nekrasov, S. Shatashvili, Integrating over Higgs branches, Commun.Math. Phys. 209 (2000) 97, arXiv:hep-th/9712241.

[16] A. Losev, N. Nekrasov, S.L. Shatashvili, Testing Seiberg–Witten solution, in: Strings, Branes and Dualities, Cargese, 1997, pp. 359–372, arXiv:hep-th/9801061.

[17] N.A. Nekrasov, Seiberg–Witten prepotential from instanton counting, Adv. Theor. Math. Phys. 7 (2004) 831, arXiv:hep-th/0206161.

[18] I. Antoniadis, I. Florakis, S. Hohenegger, K.S. Narain, A. ZeinAssi, Worldsheet realization of the refined topological string, Nucl. Phys. B 875 (2013) 101, arXiv:1302.6993 [hep-th].

[19] R. Gopakumar, C. Vafa, M theory and topological strings. 1, arXiv:hep-th/9809187; R. Gopakumar, C. Vafa, M theory and topological strings. 2,

arXiv:hep-th/9812127; T.J. Hollowood, A. Iqbal, C. Vafa, Matrix models, geometric engineering and elliptic genera, J. High Energy Phys. 0803 (2008) 069, arXiv:hep-th/0310272.

[20] H. Awata, H. Kanno, Instanton counting, Macdonald functions and the moduli space of D-branes, J. High Energy Phys. 0505 (2005) 039, arXiv:hep-th/0502061; A. Iqbal, C. Kozcaz, C. Vafa, The refined topological vertex, J. High Energy Phys. 0910 (2009) 069, arXiv:hep-th/ 0701156.

[21] R. Dijkgraaf, C. Vafa, Toda theories, matrix models, topological strings, and N = 2 gauge systems, arXiv:0909.2453 [hep-th].

[22] S. Hellerman, D. Orlando, S. Reffert, String theory of the omega deformation, J. High Energy Phys. 1201 (2012) 148, arXiv:1106.0279 [hep-th]; S. Hellerman, D. Orlando, S. Reffert, The omega deformation from string and M-theory, J. High Energy Phys. 1207 (2012) 061, arXiv:1204.4192 [hep-th].

[23] J.F. Morales, M. Serone, Higher derivative F terms in N = 2 strings, Nucl. Phys. B 481 (1996) 389, arXiv:hep-th/ 9607193.

[24] I. Antoniadis, I. Florakis, S. Hohenegger, K.S. Narain, A. ZeinAssi, Non-perturbativeNekrasov partition function from string theory, Nucl. Phys. B 880 (2014) 87, arXiv:1309.6688 [hep-th].

[25] Y. Nakayama, H. Ooguri, Comments on worldsheet description of the omega background, Nucl. Phys. B 856 (2012) 342, arXiv:1106.5503 [hep-th].

[26] A.Z. Assi, Topological amplitudes and the string effective action, EcolePolytechnique TEL-00942993, arXiv: 1402.2428 [hep-th].

[27] I. Florakis, A.Z. Assi, N = 2 string amplitudes and the omega background, arXiv:1402.2974 [hep-th].

[28] M. Billó, M. Frau, F. Fucito, A. Lerda, Instanton calculus in R–R background and the topological string, J. High Energy Phys. 0611 (2006) 012, arXiv:hep-th/0606013.

[29] K. Ito, H. Nakajima, T. Saka, S. Sasaki, N = 2 instanton effective action in -background and D3/D(-1)-brane system in R–R background, J. High Energy Phys. 1011 (2010) 093, arXiv:1009.1212 [hep-th].

[30] M.-x. Huang, A. Klemm, Direct integration for general backgrounds, Adv. Theor. Math. Phys. 16 (3) (2012) 805, arXiv:1009.1126 [hep-th].

[31] M.-x. Huang, A.-K.Kashani-Poor, A. Klemm, The deformed B-model for rigid N = 2 theories, Ann. Henri Poincaré 14 (2013) 425, arXiv:1109.5728 [hep-th].

[32] A. Klemm, M. Marino, Counting BPS states on the enriquesCalabi–Yau, Commun. Math. Phys. 280 (2008) 27, arXiv:hep-th/0512227.

[33] B. McClain, B.D.B. Roth, Modular invariance for interacting bosonic strings at finite temperature, Commun. Math. Phys. 111 (1987) 539; K.H. O'Brien, C.I. Tan, Modular invariance of thermopartition function and global phase structure of heterotic string, Phys. Rev. D 36 (1987) 1184; L.J. Dixon, V. Kaplunovsky, J. Louis, Moduli dependence of string loop corrections to gauge coupling constants, Nucl. Phys. B 355 (1991) 649.